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ORIGINAL PAPER

Comparative analysis of variational iteration method and Haar wavelet method for the numerical solutions of Burgers–Huxley and Huxley equations

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Abstract In this paper, variational iteration method is applied to compute the numerical solutions of non-linear partial differential equations like Huxley and Burgers–Huxley equation. The approximate solutions of the Huxley and Burgers–Huxley equations are compared with the Haar wavelet solution as well as with the exact solutions. The present method is very simple, effective and convenient analytical method with small computational overhead.

Keywords Variational iteration method · Haar wavelets · Huxley equation · Burgers–Huxley equation · Haar wavelet method

1 Introduction

Numerical solutions of nonlinear differential equations are of great importance in physical problems since so far there exists no general technique for finding analytical solutions of nonlinear differential equations.

Generalized Burgers–Huxley equation [1–3] is a nonlinear partial differential equation of the form

$$u_t + \alpha u^{\delta} u_x - u_{xx} = \beta u \left(1 - u^{\delta}\right)\left(u^{\delta} - \gamma\right), \quad 0 \leq x \leq 1, t \geq 0 \tag{1.1}$$

where α, β, γ and δ are parameters, $\beta \geq 0$, $\gamma, \delta > 0$. When $\alpha = 0, \delta = 1$, Eq. (1.1) reduces to the Huxley equation. The Huxley equation [4–6] is a nonlinear partial differential equation of second order of the form

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$$u_t = u_{xx} + u\,(k-u)\,(u-1)\,,\, k \neq 0 \tag{1.2}$$

This equation is an evolution equation that describes nerve pulse propagation in biology from which molecular CB properties can be calculated. Generalized Burgers–Huxley equation is of high importance for describing the interaction between reaction mechanisms, convection effects, and diffusion transport.

Various powerful mathematical methods such as Adomian decomposition method [7,1], spectral collocation method [2], the tanh-coth method [8], homotopy perturbation method [5], Exp-Function method [6] and Differential Quadrature method [3] have been used in attempting to solve the Burgers–Huxley and the Huxley equations. The solitary wave solutions of the generalized Burgers–Huxley equation have been studied by the learned researchers [9,10].

The variational iteration method (VIM) was developed by He in [4,11,12]. The method is now used by many researchers in a variety of scientific fields. The method has been proved by many authors to be reliable and efficient for a variety of scientific applications, linear and nonlinear as well. The method gives the solution in the form of rapidly convergent successive approximations that may give the exact solution if such a solution exists. For concrete problems where exact solution is not obtainable, it was found that a few numbers of approximations can be used for numerical purposes.

In this present paper, Wavelet collocation method is also used for solving generalized Burger–Huxley and Huxley equations. This method consists of reducing the problem to a set of algebraic equation by expanding the term, which has maximum derivative, given in the equation as Haar functions with unknown coefficients. The operational matrix of integration and product operational matrix are utilized to evaluate the coefficients of Haar functions. This method gives us the implicit form of the approximate solutions of the problems.

This paper is systematized as follows: in Sect. 1, introduction to Burgers–Huxley and Huxley equation is described. In Sect. 2, the mathematical preliminaries of Variational iteration method are presented. Section 3 represents the mathematical preliminaries of Haar wavelets. Sections 4 and 7 define the mathematical models of Huxley and Burgers–Huxley equation respectively. We applied the Variational iteration method to Huxley and Burgers–Huxley equation in Sects. 5 and 8 respectively. We applied the Haar wavelet method to Huxley and Burgers–Huxley equation in Sects. 6 and 9 respectively. The numerical results and discussions are discussed in Sects. 10 and 11 concludes the paper.

2 Variational iteration method

To illustrate the basic concepts of the Variational iteration method, we consider the following differential equation

$$Lu + Nu = g\,(x, t) \tag{2.1}$$

where L is a linear operator, N a nonlinear operator and $g\,(x, t)$ an inhomogeneous term. According to the variational iteration method, we can construct a correction functional as follows

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda\left\{Lu_n(x,t)+N\widetilde{u_n}(x,t)-g(x,t)\right\}dt, n\geq 0, \quad (2.2)$$

where λ is a general Lagrangian multiplier which can be identified optimally by the variational theory, the subscript n denotes the nth order approximation, and $N\widetilde{u_n}(x,t)$ is considered as a restricted variation, i.e. $\delta N\widetilde{u_n}(x,t) = 0$.

3 Haar wavelets and the operational matrices

The Haar wavelet family for $x \in [0, 1)$ is defined as follows [13,14]

$$h_i(x) = \begin{cases} 1 & x \in [\xi_1, \xi_2) \\ -1 & x \in [\xi_2, \xi_3) \\ 0 & \text{elsewhere} \end{cases} \quad (3.1)$$

where

$$\xi_1 = \frac{k}{m}, \quad \xi_2 = \frac{k+0.5}{m}, \quad \xi_3 = \frac{k+1}{m}.$$

In these formulae integer $m = 2^j$, $j = 0, 1, 2, \ldots, J$ indicates the level of the wavelet; $k = 0, 1, 2, \ldots, m-1$ is the translation parameter. Maximum level of resolution is J. The index i is calculated from the formula $i = m + k + 1$; in the case of minimal values $m = 1, k = 0$ we have $i = 2$. The maximal value of $i = 2M = 2^{J+1}$. It is assumed that the value $i = 1$ corresponds to the scaling function for which

$$h_i(x) = \begin{cases} 1 \text{ for } x \in [0, 1) \\ 0 \text{ elsewhere} \end{cases} \quad (3.2)$$

In the following analysis, integrals of the wavelets are defined as

$$p_i(x) = \int_0^x h_i(x)dx, \quad q_i(x) = \int_0^x p_i(x)dx \quad (3.3)$$

The collocation points are defined as

$$x_l = \frac{l-0.5}{2M}, \quad l = 1, 2, \ldots, 2M$$

It is expedient to introduce the $2M \times 2M$ matrices H, P, Q with the elements $H(i,l) = h_i(x_l)$, $P(i,l) = p_i(x_l)$, $Q(i,l) = q_i(x_l)$.

4 Huxley equation

Huxley equation is a nonlinear partial differential equation of second order in the form

$$u_t = u_{xx} + u\,(k-u)\,(u-1) \tag{4.1}$$

with initial condition

$$u\,(x,0) = \frac{1}{2}\left[1+\tanh\left(\frac{x}{2\sqrt{2}}\right)\right] \tag{4.2}$$

The exact solution of Eq. (4.1) is given by [6]

$$u\,(x,t) = \frac{1}{2}\left[1+\tanh\left\{\frac{1}{2\sqrt{2}}\left(x-\frac{2k-1}{\sqrt{2}}t\right)\right\}\right], \quad k \neq 0 \tag{4.3}$$

Taking $k = 1$, the boundary conditions are

$$\begin{aligned} u\,(0,t) &= \frac{1}{2}\left[1-\tanh\left(\frac{t}{4}\right)\right] \\ u\,(1,t) &= \frac{1}{2}\left[1+\tanh\left\{\frac{1}{2\sqrt{2}}\left(1-\frac{t}{\sqrt{2}}\right)\right\}\right] \end{aligned} \tag{4.4}$$

5 Application of variational iteration method for solving Huxley equation

Let us construct a correction functional as follows

$$u_{n+1}\,(x,t) = u_n\,(x,t) + \int_0^t \lambda\left\{(u_n)_\tau - (\widetilde{u_n})_{xx} - u_n\,(k-\widetilde{u_n})\,(u_n-1)\right\}d\tau \tag{5.1}$$

where λ is a general Lagrangian multiplier whose optimal value can be found using variational theory and $\tilde{u}_n\,(x,t)$ is the restricted variation, i.e. $\delta\tilde{u}_n\,(x,t) = 0$.

$$\delta u_{n+1}\,(x,t) = \delta u_n\,(x,t) + \delta\int_0^t \lambda\left\{(u_n)_\tau - (\widetilde{u_n})_{xx} - u_n\,(k-\widetilde{u_n})\,(u_n-1)\right\}d\tau \tag{5.2}$$

From the above Eq. (5.2), we have

$$\delta u_{n+1}\,(x,t) = \delta u_n\,(x,t) + \delta\int_0^t \lambda\left(\frac{\partial u_n}{\partial\tau}\right)d\tau \tag{5.3}$$

since $\delta \left(\widetilde{u_n}\right)_{xx} = 0$ and $\delta \left(u_n \left(k - \widetilde{u_n}\right)(u_n - 1)\right) = 0$

Integrating right hand side of Eq. (5.3) yields

$$\delta u_{n+1} = \delta u_n + \left(\lambda|_{\tau=t}\right) \delta u_n - \int_0^t \lambda' u_n d\tau \tag{5.4}$$

From stationary condition, we know $\delta u_{n+1} = 0$

which yields that

$$\lambda'(\tau) = 0 \text{ and } 1 + \lambda|_{\tau=t} = 0 \tag{5.5}$$

$\Rightarrow \lambda = \text{c}$. Hence $\lambda = -1$

Therefore the following variational iteration formula can be obtained as

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left\{(u_n)_\tau - (u_n)_{xx} - u_n (k - u_n)(u_n - 1)\right\} d\tau \tag{5.6}$$

From the above iteration formula Eq. (5.6) we can obtained

$$u_1(x,t) = 0.5 + 0.5 \tanh\left(\frac{x}{2\sqrt{2}}\right) - 0.125\, t \,\mathrm{Sech}^2\left(\frac{x}{2\sqrt{2}}\right) \tag{5.7}$$

with initial condition

$$u_0(x,t) = \frac{1}{2}\left[1 + \tanh\left(\frac{x}{2\sqrt{2}}\right)\right]$$

6 Application of Haar wavelet method for solving Huxley equation

It is assumed that $\dot{u}''(x,t)$ can be expanded in terms of Haar wavelets as

$$\dot{u}''(x,t) = \sum_{i=1}^{2M} a_s(i)\, h_i(x) \text{ for } t \in \left[t_s, t_{s+1}\right] \tag{6.1}$$

where "**.**" and "′" stands for differentiation with respect to t and x respectively.

Now, integrating Eq. (6.1) with respect to t from t_s to t and twice with respect to x from 0 to x the following equations are obtained

$$u''(x,t) = (t - t_s) \sum_{i=1}^{2M} a_s(i)\, h_i(x) + u''(x, t_s)$$

$$u'(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)\, p_i(x) + u'(x,t_s) - u'(0,t_s) + u'(0,t) \tag{6.2}$$

$$\begin{aligned} u(x,t) &= (t-t_s)\sum_{i=1}^{2M} a_s(i)\, q_i(x) + u(x,t_s) - u(0,t_s) \\ &\quad + x\left[u'(0,t) - u'(0,t_s)\right] + u(0,t) \\ \dot{u}(x,t) &= \sum_{i=1}^{2M} a_s(i)\, q_i(x) + x\dot{u}'(0,t) + \dot{u}(0,t) \end{aligned}$$

By using the boundary conditions, at $x = 1$, we have

$$\dot{u}'(0,t) = -\sum_{i=1}^{2M} a_s(i)\, q_i(1) + \dot{u}(1,t) - \dot{u}(0,t) \tag{6.3}$$

From Eq. (3.3), it is obtained that,

$$q_i(1) = \begin{cases} 0.5 & \text{if } i = 1 \\ \frac{1}{4\text{m}^2} & \text{if } i > 1 \end{cases} \tag{6.4}$$

Discretising the results by assuming $x \to x_l$, $t \to t_{s+1}$, and substituting along with Eqs. (6.2), (6.3) and (6.4) in Eq. (4.1), we have

$$\begin{aligned} \sum_{i=1}^{2M} a_s(i)\left[q_i(x_l) - x_l q_i(1)\right] &= u''(x_l,t_s) - u(x_l,t_s)\left[1-u(x_l,t_s)\right]^2 \\ &\quad - \dot{u}(0,t_{s+1}) - x_l\left[\dot{u}(1,t_{s+1}) - \dot{u}(0,t_{s+1})\right] \end{aligned} \tag{6.5}$$

From Eq. (6.5), the wavelet coefficients $a_s(i)$ can be successively calculated. This process started with

$$\begin{aligned} u(x_l,t_0) &= \frac{1}{2}\left[1 + \tanh\left(\frac{x_l}{2\sqrt{2}}\right)\right] \\ u'(x_l,t_0) &= \frac{1}{4\sqrt{2}}\sec h^2\left(\frac{x_l}{2\sqrt{2}}\right) \\ u''(x_l,t_0) &= -\frac{1}{8}\sec h^2\left(\frac{x_l}{2\sqrt{2}}\right)\tanh\left(\frac{x_l}{2\sqrt{2}}\right) \end{aligned}$$

7 Burgers–Huxley equation

Consider the generalized Burgers–Huxley equation

$$u_t + \alpha u^\delta u_x - u_{xx} = \beta u\left(1-u^\delta\right)\left(u^\delta - \gamma\right), \quad 0 \le x \le 1, t \ge 0 \tag{7.1}$$

with initial condition

$$u\,(x,0)=\left(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh\left[A_1x\right]\right)^{1/\delta} \tag{7.2}$$

The exact solution of Eq. (7.1) is given by [2]

$$u\,(x,t)=\left(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh\left[A_1\,(x-A_2t)\right]\right)^{1/\delta} \tag{7.3}$$

where

$$A_1=\frac{-\alpha\delta+\delta\sqrt{\alpha^2+4\beta\,(1+\delta)}}{4\,(1+\delta)}\gamma,$$
$$A_2=\frac{\gamma\alpha}{(1+\delta)}-\frac{(1+\delta-\gamma)\left(-\alpha+\sqrt{\alpha^2+4\beta\,(1+\delta)}\right)}{2\,(1+\delta)}, \tag{7.4}$$

where α, β, γ and δ are parameters with $\beta\geq 0$ and $\delta>0$.

This exact solution satisfies the following boundary conditions

$$u\,(0,t)=\left[\frac{\gamma}{2}+\frac{\gamma}{2}\tanh\left(-A_1A_2t\right)\right]^{1/\delta},\,t\geq 0$$
$$u\,(1,t)=\left[\frac{\gamma}{2}+\frac{\gamma}{2}\tanh\left(A_1\,(1-A_2t)\right)\right]^{1/\delta},\,t\geq 0 \tag{7.5}$$

8 Application of variational iteration method for solving Burgers–Huxley equation

Construct a correction functional as follows

$$u_{n+1}\,(x,t)=u_n\,(x,t)+\int_0^t\lambda\left\{(u_n)_\tau-\widetilde{\left(u_n^\delta\right)}\,(u_n)_x-\widetilde{(u_n)}_{xx}-u_n\,(1-u_n)\,\widetilde{(u_n}-0.001)\right\}d\tau \tag{8.1}$$

where λ is a general Lagrangian multiplier whose optimal value can be found using variational theory and $\tilde{u}_n\,(x,t)$ is the restricted variation, i.e. $\delta\tilde{u}_n\,(x,t)=0$.

$$\delta u_{n+1}\,(x,t)=\delta u_n\,(x,t)+\delta\int_0^t\lambda\left\{(u_n)_\tau-\widetilde{\left(u_n^\delta\right)}\,(u_n)_x-\widetilde{(u_n)}_{xx}\right.$$
$$\left.-\,u_n\,(1-u_n)\,\widetilde{(u_n}-0.001)\right\}d\tau \tag{8.2}$$

From the above Eq. (8.2), we have

$$\delta u_{n+1}(x,t)=\delta u_n(x,t)+\delta\int_0^t \lambda\left(\frac{\partial u_n}{\partial\tau}\right)d\tau, \tag{8.3}$$

since $\delta\left(\left(u_n^{\delta}\right)\widetilde{(u_n)_x}\right)=0$, $\delta\,\widetilde{(u_n)}_{xx}=0$ and $\delta\left(u_n(1-u_n)\widetilde{(u_n-0.001)}\right)=0$

Integrating right hand side of Eq. (8.3) yields

$$\delta u_{n+1}=\delta u_n+\left(\lambda|_{\tau=t}\right)\delta u_n-\int_0^t \lambda' u_n d\tau \tag{8.4}$$

From stationary condition we know $\delta u_{n+1}=0$

which yields that

$$1+\lambda|_{\tau=t}=0 \text{ and } \lambda'(t)=0 \tag{8.5}$$
$$\Rightarrow \lambda=\mathrm{c}$$

Hence $\lambda=-1$

Therefore λ can be identified as -1, and the following variational iteration formula can be obtained as

$$u_{n+1}(x,t)=u_n(x,t)-\int_0^t \left\{(u_n)_\tau-(u_n)(u_n)_x-(u_n)_{xx}-u_n(1-u_n)(u_n-0.001)\right\}d\tau \tag{8.6}$$

From the above iteration formula Eq. (8.6), we can obtained

$$u_1(x,t)=0.0005+0.0005\tanh(0.00025\,x)-0.000000249938\,t\ \mathrm{Sech}^2(0.00025\,x) \tag{8.7}$$

with initial condition

$$u_0(x,t)=0.0005+0.0005\tanh(0.00025x)$$

9 Application of Haar wavelet method for solving Burgers–Huxley equation

Haar wavelet solution of $u(x,t)$ is sought by assuming that $\dot{u}''(x,t)$ can be expanded in terms of Haar wavelets as

$$\dot{u}''(x,t)=\sum_{i=1}^{2M} a_s(i)\,h_i(x) \text{ for } t\in\left[t_s,t_{s+1}\right] \tag{9.1}$$

where "." and " ′ " stands for differentiation with respect to t and x respectively.

Integrating Eq. (9.1) with respect to t from t_s to t and twice with respect to x from 0 to x the following equations are obtained

$$u''(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)h_i(x) + u''(x,t_s)$$
$$u'(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)p_i(x) + u'(x,t_s) - u'(0,t_s) + u'(0,t) \tag{9.2}$$
$$u(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i)q_i(x) + u(x,t_s) - u(0,t_s) + x[u'(0,t) - u'(0,t_s)] + u(0,t)$$
$$\dot{u}(x,t) = \sum_{i=1}^{2M} a_s(i)\, q_i(x) + x\dot{u}'(0,t) + \dot{u}(0,t)$$

By using the boundary conditions, at $x = 1$, we have

$$\dot{u}'(0,t) = -\sum_{i=1}^{2M} a_s(i)\, q_i(1) + \dot{u}(1,t) - \dot{u}(0,t) \tag{9.3}$$

It is obtained from Eq. (3.3) that,

$$q_i(1) = \begin{cases} 0.5 & \text{if } i = 1 \\ \frac{1}{4\mathrm{m}^2} & \text{if } i > 1 \end{cases} \tag{9.4}$$

Discretising the results by assuming $x \to x_l, t \to t_{s+1}$ and substituting along with Eqs. (9.2), (9.3) and (9.4) in Eq. (7.1), we have

$$\begin{aligned}\sum_{i=1}^{2M} a_s(i)\,[q_i(x_l) - x_l q_i(1)] &= u''(x_l,t_s) + u(x_l,t_s)\,[1 - u(x_l,t_s)] \\ &\times [u(x_l,t_s) - 0.001] - u(x_l,t_s)\,u'(x_l,t_s) - \dot{u}(0,t_{s+1}) \\ &- x_l\left[\dot{u}(1,t_{s+1}) - \dot{u}(0,t_{s+1})\right]\end{aligned} \tag{9.5}$$

From the above equation the wavelet coefficients $a_s(i)$ can be successively calculated. This process started with

$$u(x_l,t_0) = 1 + \tanh\left(\frac{x_l}{2}\right)$$
$$u'(x_l,t_0) = \frac{1}{2}\sec h^2\left(\frac{x_l}{2}\right)$$
$$u''(x_l,t_0) = -\frac{1}{2}\sec h^2\left(\frac{x_l}{2}\right)\tanh\left(\frac{x_l}{2}\right)$$

Table 1 The absolute errors for the solutions of Burgers–Huxley equation using Haar wavelet method and one iteration of VIM at various collocation points for x with $t = 0.4$ and $\gamma = 0.001$

x	Approximate solutions using Haar wavelet method (u_{approx})	Approximate solutions using VIM (u_{approx})	Exact solutions (u_{exact})	Absolute errors using Haar wavelet method	Absolute errors using VIM
0.03125	0.00050006	0.00049994	0.000500054	6.5661e−9	1.49925e−7
0.09375	0.000500121	0.000499912	0.000500062	5.90949e−8	1.49925e−7
0.15625	0.000500234	0.00049992	0.000500069	1.64153e−7	1.49925e−7
0.21875	0.000500399	0.000499927	0.000500077	3.21739e−7	1.49925e−7
0.28125	0.000500617	0.000499935	0.000500085	5.31854e−7	1.49925e−7
0.34375	0.000500887	0.000499943	0.000500093	7.94498e−7	1.49925e−7
0.40625	0.00050121	0.000499951	0.000500101	1.10967e−6	1.49925e−7
0.46875	0.000501586	0.000499959	0.000500109	1.47737e−6	1.49925e−7
0.53125	0.000502014	0.000499966	0.000500116	1.8976e−6	1.49925e−7
0.59375	0.000502495	0.000499974	0.000500124	2.37036e−6	1.49925e−7
0.65625	0.000503028	0.000499982	0.000500132	2.89565e−6	1.49925e−7
0.71875	0.000503613	0.00049999	0.00050014	3.47347e−6	1.49925e−7
0.78125	0.000504251	0.000499998	0.000500148	4.10381e−6	1.49925e−7
0.84375	0.000504942	0.000500005	0.000500155	4.78669e−6	1.49925e−7
0.90625	0.000505685	0.000500013	0.000500163	5.5221e−6	1.49925e−7
0.96875	0.000506481	0.000500021	0.000500171	6.31009e−6	1.49925e−7

10 Numerical results and discussions

The following tables show the comparisons of the exact solutions with the approximate solutions of Burgers–Huxley equation taking $\alpha = 1, \beta = 1, \gamma = 0.001, \delta = 1$ and different values of t. In Tables 1, 2 and 3, J is taken as 3 i.e. $M = 8$ and Δt is taken as 0.0001.

The error function is given by

$$\begin{aligned} Error\ function &= \left\| u_{approx}(x_l, t) - u_{exact}(x_l, t) \right\| \\ &= \sqrt{\sum_{l=1}^{2M} \left(u_{approx}(x_l, t) - u_{exact}(x_l, t) \right)^2} \\ Global\ error\ estimate = R.M.S.\ error &= \frac{\left\| u_{approx}(x_l, t) - u_{exact}(x_l, t) \right\|}{\sqrt{2M}} \\ &= \frac{1}{\sqrt{2M}} \sqrt{\sum_{l=1}^{2M} \left(u_{approx}(x_l, t) - u_{exact}(x_l, t) \right)^2} \end{aligned} \tag{10.1}$$

In case of $\gamma = 0.001$, the *R.M.S. error* between the Haar wavelet solutions and the exact solutions of Burgers–Huxley equations for $t = 0.4, 0.6$ and 1 are 0.00000300204,

Table 2 The absolute errors for the solutions of Burgers–Huxley equation using Haar wavelet method and one iteration of VIM at various collocation points for x with $t = 0.6$ and $\gamma = 0.001$

x	Approximate solutions using Haar wavelet method (u_{approx})	Approximate solutions using VIM (u_{approx})	Exact solutions (u_{exact})	Absolute errors using Haar wavelet method	Absolute errors using VIM
0.03125	0.000500089	0.000499854	0.000500079	9.84903e−9	2.24888e−7
0.09375	0.000500175	0.000499862	0.000500087	8.86412e−8	2.24888e−7
0.15625	0.000500341	0.00049987	0.000500094	2.46226e−7	2.24888e−7
0.21875	0.000500585	0.000499877	0.000500102	4.82602e−7	2.24888e−7
0.28125	0.000500908	0.000499885	0.00050011	7.97771e−7	2.24888e−7
0.34375	0.00050131	0.000499893	0.000500118	1.19173e−6	2.24888e−7
0.40625	0.00050179	0.000499901	0.000500126	1.664e−6	2.24888e−7
0.46875	0.00050235	0.000499909	0.000500134	2.216e−6	2.24888e−7
0.53125	0.000502988	0.000499916	0.000500141	2.847e−6	2.24888e−7
0.59375	0.000503705	0.000499924	0.000500149	3.556e−6	2.24888e−7
0.65625	0.0005045	0.000499932	0.000500157	4.343e−6	2.24888e−7
0.71875	0.000505375	0.00049994	0.000500165	5.21e−6	2.24888e−7
0.78125	0.000506328	0.000499948	0.000500173	6.155e−6	2.24888e−7
0.84375	0.00050736	0.000499956	0.00050018	7.18e−6	2.24888e−7
0.90625	0.000508471	0.000499963	0.000500188	8.283e−6	2.24888e−7
0.96875	0.000509661	0.000499971	0.000500196	9.465e−6	2.24888e−7

Table 3 The absolute errors for the solutions of Burgers–Huxley equation using Haar wavelet method and one iteration of VIM at various collocation points for x with $t = 1$ and $\gamma = 0.001$

x	Approximate solutions using Haar wavelet method (u_{approx})	Approximate solutions using VIM (u_{approx})	Exact solutions (u_{exact})	Absolute Errors using Haar wavelet method	Absolute Errors using VIM
0.03125	0.000500145	0.000499754	0.000500129	1.6414e−8	3.74813e−7
0.09375	0.000500284	0.000499762	0.000500137	1.47726e−7	3.74813e−7
0.15625	0.000500555	0.00049977	0.000500144	4.10351e−7	3.74813e−7
0.21875	0.000500957	0.000499777	0.000500152	8.04288e−7	3.74813e−7
0.28125	0.00050149	0.000499785	0.00050016	1.32954e−6	3.74813e−7
0.34375	0.000502154	0.000499793	0.000500168	1.9861e−6	3.74813e−7
0.40625	0.00050295	0.000499801	0.000500176	2.774e−6	3.74813e−7
0.46875	0.000503877	0.000499809	0.000500183	3.694e−6	3.74813e−7
0.53125	0.000504935	0.000499816	0.000500191	4.744e−6	3.74813e−7
0.59375	0.000506125	0.000499824	0.000500199	5.926e−6	3.74813e−7
0.65625	0.000507445	0.000499832	0.000500207	7.238e−6	3.74813e−7
0.71875	0.000508898	0.00049984	0.000500215	8.683e−6	3.74813e−7
0.78125	0.000510481	0.000499848	0.000500223	1.0258e−5	3.74813e−7
0.84375	0.000512196	0.000499856	0.00050023	1.1966e−5	3.74813e−7
0.90625	0.000514042	0.000499863	0.000500238	1.3804e−5	3.74813e−7
0.96875	0.00051602	0.000499871	0.000500246	1.5774e−5	3.74813e−7

Table 4 The absolute errors for the solutions of Huxley equation using Haar wavelet method and one iteration of VIM at various collocation points for x with $k = 1$ and $t = 0.4$

x	Approximate solutions using Haar wavelet method (u_{approx})	Approximate solutions using VIM (u_{approx})	Exact solutions (u_{exact})	Absolute errors using Haar wavelet	Absolute errors using VIM
0.03125	0.455737	0.45553	0.455641	0.0000955529	0.000111054
0.09375	0.467153	0.466622	0.466623	0.000530592	8.87183e−7
0.15625	0.478927	0.477746	0.477636	0.0012907	0.000109283
0.21875	0.491048	0.488891	0.488672	0.00237582	0.000219028
0.28125	0.503505	0.500046	0.499718	0.00378609	0.000327923
0.34375	0.516287	0.511201	0.510765	0.00552185	0.000435548
0.40625	0.529385	0.522343	0.521802	0.0075836	0.000541497
0.46875	0.542789	0.533462	0.532817	0.00997205	0.000645376
0.53125	0.556488	0.544547	0.5438	0.012688	0.000746806
0.59375	0.570474	0.555586	0.554741	0.0157326	0.00084543
0.65625	0.584736	0.56657	0.565629	0.019107	0.000940912
0.71875	0.599266	0.577487	0.576454	0.0228126	0.00103294
0.78125	0.614057	0.588327	0.587206	0.026851	0.00112123
0.84375	0.629099	0.599081	0.597876	0.031223	0.00120552
0.90625	0.645576	0.609739	0.608453	0.037123	0.00128559
0.96875	0.668244	0.620291	0.61893	0.049314	0.00136124

Table 5 The absolute errors for the solutions of Huxley equation using Haar wavelet method and one iteration of VIM at various collocation points for x with $k = 1$ and $t = 0.6$

x	Approximate solutions using Haar wavelet method (u_{approx})	Approximate solutions using VIM (u_{approx})	Exact solutions (u_{exact})	Absolute errors using Haar wavelet	Absolute errors using VIM
0.03125	0.431155	0.430533	0.430968	0.000187446	0.000434778
0.09375	0.442789	0.441649	0.441837	0.000951983	0.000188225
0.15625	0.454998	0.452822	0.452763	0.00223552	0.0000590574
0.21875	0.467771	0.46404	0.463734	0.00403766	0.000306109
0.28125	0.481098	0.475292	0.47474	0.0063583	0.000551971
0.34375	0.494968	0.486567	0.485771	0.0091976	0.000795698
0.40625	0.509372	0.497852	0.496816	0.012556	0.00103636
0.46875	0.524298	0.509136	0.507863	0.0164343	0.00127304
0.53125	0.539737	0.520408	0.518904	0.0208334	0.00150489
0.59375	0.55568	0.531656	0.529925	0.0257546	0.00173105
0.65625	0.572117	0.542869	0.540918	0.0311994	0.00195074
0.71875	0.589041	0.554034	0.551871	0.0371696	0.00216323
0.78125	0.606441	0.565142	0.562774	0.043667	0.00236782
0.84375	0.62431	0.57618	0.573616	0.050694	0.00256389
0.90625	0.645842	0.58714	0.584389	0.061453	0.00275089
0.96875	0.683829	0.598009	0.595081	0.088748	0.00292832

Table 6 The absolute errors for the solutions of Huxley equation using Haar wavelet method and one iteration of VIM at various collocation points for x with $k = 1$ and $t = 1$

x	Approximate solutions using Haar wavelet method (u_{approx})	Approximate solutions using VIM (u_{approx})	Exact solutions (u_{exact})	Absolute errors using Haar wavelet	Absolute errors using VIM
0.03125	0.383171	0.380539	0.382747	0.00042376	0.00220814
0.09375	0.395066	0.391704	0.393241	0.00182475	0.00153709
0.15625	0.407796	0.402974	0.403834	0.00396201	0.000860175
0.21875	0.421352	0.414338	0.414518	0.00683392	0.000179977
0.28125	0.435721	0.425783	0.425282	0.0104393	0.000500891
0.34375	0.450895	0.437298	0.436118	0.0147773	0.00117981
0.40625	0.466863	0.448869	0.447015	0.0198477	0.00185419
0.46875	0.483614	0.460485	0.457964	0.0256507	0.00252146
0.53125	0.501139	0.472132	0.468953	0.0321868	0.00317912
0.59375	0.519429	0.483797	0.479972	0.0394572	0.00382474
0.65625	0.538474	0.495467	0.491011	0.0474633	0.00445598
0.71875	0.558265	0.507129	0.502058	0.056207	0.00507062
0.78125	0.578795	0.51877	0.513104	0.065691	0.00566659
0.84375	0.600054	0.530378	0.524137	0.075917	0.00624192
0.90625	0.632639	0.541941	0.535146	0.097493	0.00679482
0.96875	0.718961	0.553445	0.546121	0.17284	0.00732368

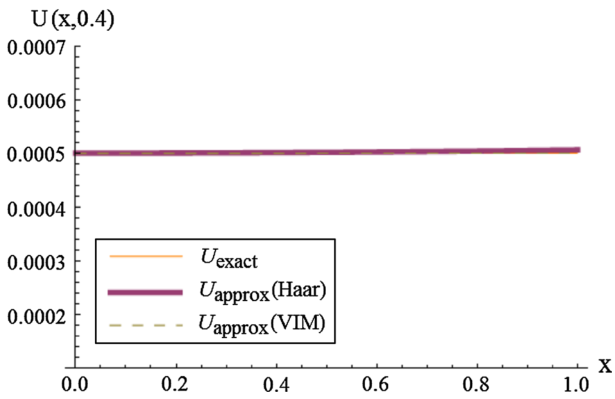

Fig. 1 Comparison of Haar wavelet solutions and VIM solutions with the exact solution of Burgers–Huxley equation when $t = 0.4$ and $\gamma = 0.001$

0.00000450295 and 0.00000750449 respectively and that of the VIM solutions and the exact solutions are 0.000000149925, 0.000000224888 and 0.000000374813 respectively.

In the following Tables 4, 5 and 6 also J has been taken as 3 i.e. $M = 8$ and Δt is taken as 0.0001. Again, the *R.M.S. error* can be calculated from Eq. (10.1) for

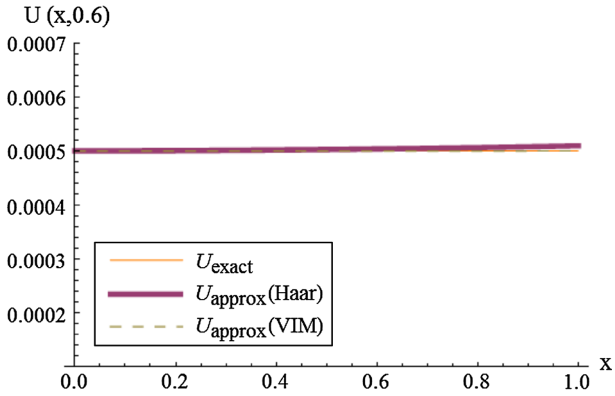

Fig. 2 Comparison of Haar wavelet solutions and VIM solutions with the exact solution of Burgers–Huxley equation when $t = 0.6$ and $\gamma = 0.001$

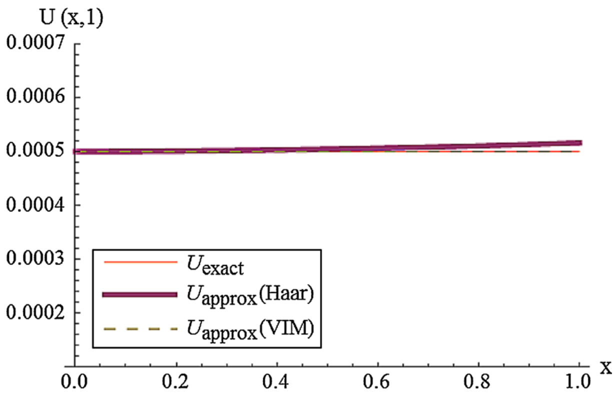

Fig. 3 Comparison of Haar wavelet solutions and VIM solutions with the exact solution of Burgers–Huxley equation when $t = 1$ and $\gamma = 0.001$

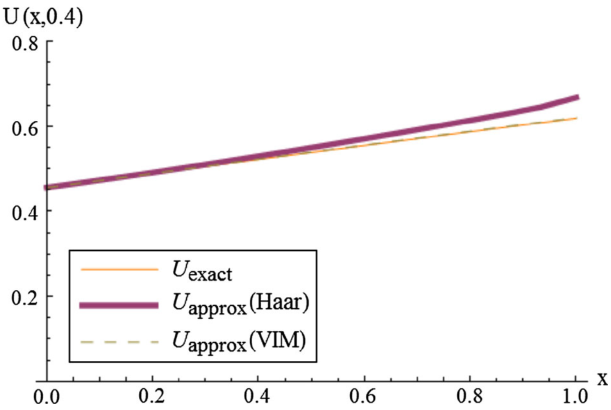

Fig. 4 Comparison of Haar wavelet solutions and VIM solutions with the exact solution of Huxley equations when $t = 0.4$ and $k = 1$

different values of t. For $t = 0.4, 0.6$ and 1, the *R.M.S. error* between the Haar wavelet solutions and the exact solutions of Huxley equation are 0.0209303, 0.0354936 and 0.060677 respectively.

For $t = 0.4, 0.6$ and 1, the *R.M.S. error* between the VIM solutions and the exact solutions of Huxley equation are 0.000810868, 0.001696 and 0.00403601 respectively.

In case of Burgers–Huxley equation, the Figs. 1, 2 and 3 cite the comparison graphically between the numerical solutions obtained by Haar wavelet method, VIM and exact solutions for different values of t and γ. Similarly, in case of Huxley equation, the Fig. 4 present the comparison graphically between the numerical results obtained by Haar wavelet method, VIM and exact solutions for different values of t and $k = 1$.

11 Conclusions

In this paper, the generalized Burgers–Huxley equation and Huxley equation have been solved by Variational Iteration method and by Haar wavelet method. The obtained results are then compared with exact solutions. These results cited in the tables and also graphically demonstrate the comparison of Variational Iteration method and Haar wavelet method. The main advantage of this method is its simplicity and small computational overhead.

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